Conclusions

Results of the present experiments show increasing plateau pressure and decreasing length of the interaction region with decreasing Reynolds number and decreasing wall-to-recovery temperature ratio. Values of plateau pressure coefficient were proportional to the square root of the skin-friction coefficient of the undisturbed boundary layer, a scaling law derived from Chapman's formulation of free interaction, and maximum static pressure gradients were found to agree reasonably well with Zukoski's correlation of adiabatic data.

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Direct Measurement of the Velocity Gradient in a Fluid Flow

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Introduction

THE usual technique of constructing the velocity gradient in a fluid flow is to measure the velocity profile and then numercally or graphically differentiate the data. This technique is inherently inaccurate because it is an averaging technique. Points of inflection and large gradients in the velocity profile are either inaccurately measured or not detected.

In this research, a fundamental extension of the hot-wire technique was used to measure directly the gradient of the mean velocity profile. Basically, the method requires the use of a vibrating hot-wire anemometer. It will be shown that the oscillatory anemometer response is proportional to the velocity gradient in the flow. Detection of the oscillatory signal of the hot-wire at the frequency of forced oscillation with a Lock-In Amplifier (LIA) gives the direct measurement of the velocity gradient.

The technique has direct application to the diagnosis of stratified flows, the fluid mechanics of the air-sea interface, and the phenomeonological description of turbulence.

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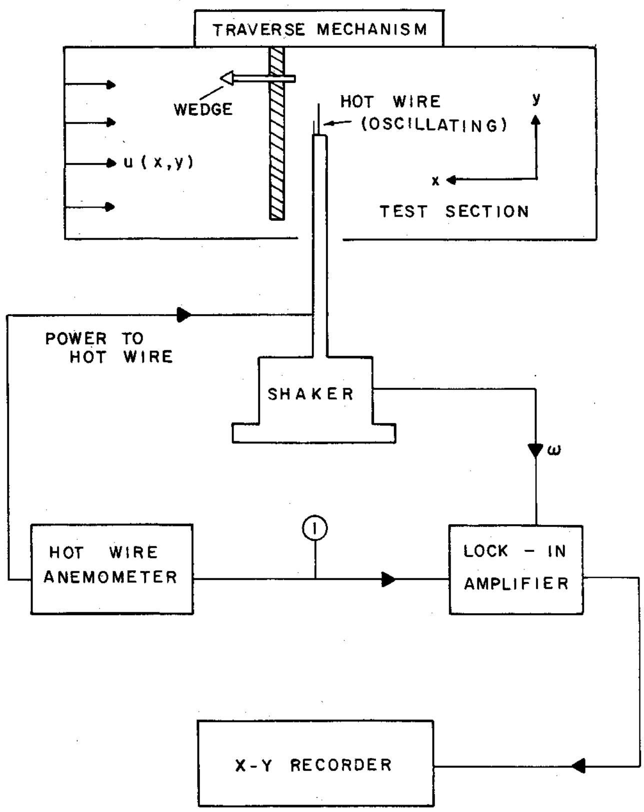

Fig. 1 Experimental schematic.

Theory

Figure 1 is a schematic of a hot wire probe oscillating in a fluid flow of mean velocity $u(x, y)$. The probe is constrained to oscillate in the y-z plane perpendicular to the velocity $u(x, y)$. The wire performs simple harmonic motion according to the relation

$$y(t) = y_0 + R\sin\omega t \tag{1}$$

where R is the amplitude of the oscillation at a fixed point x_0, and ω is the radian frequency of forced oscillation. The velocity V as seen by the probe may be written as

$$V = [u(x, y)^2 + (\omega R)^2 \cos^2 \omega t]^{1/2} \tag{2}$$

Suppressing the argument x, the velocity $u^2(y)$ may be expanded in a Taylor series about y_0 and combined with Eq. (1) and (2) to form the following relation for the velocity V as seen by the probe

$$V^2(t) = u^2 + (R^2/2)(uu'' + u'^2 + \omega^2) + \sin\omega t\{2uu'R + (R^3/4)(uu''' + 3u'u'')\} + \cos 2\omega t\{(\omega R)^2/2 - R^2u'^2/2 - R^2uu''/2\} + \sin 3\omega t\{-R^3/12(uu''' + 3u'u'')\} + \ldots \tag{3}$$

Let δ be the scale length of the gradient of u in the y direction. Define a dimensionless distance $y^* = y/\delta$ and a dimensionless velocity $u^* = u/U_\infty$. Equation (3) may then be written as

$$V^2(t) = U_\infty^2\{u^{*2} + \tfrac{1}{2}(R/\delta)^2 (u^*u''^* + u'^{*2} + \omega^2) + [2u^*u'^*R/\delta + (R/\delta)^3 (u^*u'''^*/4 - \tfrac{3}{4}u'^*u''^*)]\sin\omega t + [\omega^2/2(R/\delta)^2 - (R/\delta)^2 u'^{*2}/2 - (R/\delta)^2 u^*u''^*/2] \cos 2\omega t + [-(R/\delta)^3 (u^*u'''^*/12 + u'^*u''^*/4)]\sin 3\omega t + \ldots\} \tag{4}$$

If R/δ can be made $<< 1$ and consideration is given to the coefficient of $\sin\omega t$ then Eq. (3) may be written approximately as

$$V^2(t) \simeq 2uu'R\sin\omega t \tag{5}$$

Similarly if it is desired to consider terms at frequency 2ω, then Eq. (3) may be written approximately as

$$V^2(t) = (R^2/2)[\omega^2 - u'^2 - uu'']\cos 2\omega t \tag{6}$$

If the oscillatory amplitude of $V^2(t)$ could be measured at the forcing frequency, ω or 2ω, and a known amplitude of oscillation,

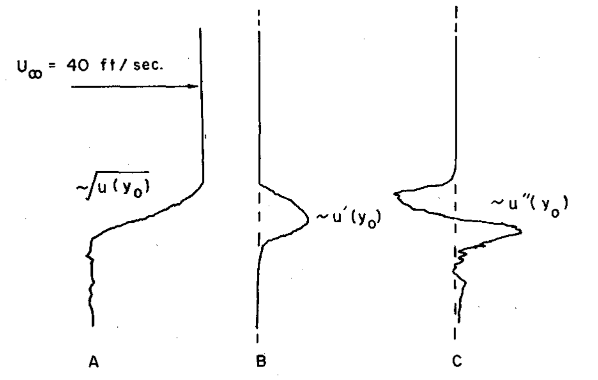

Fig. 2 *X-Y* **Recorder traces.**

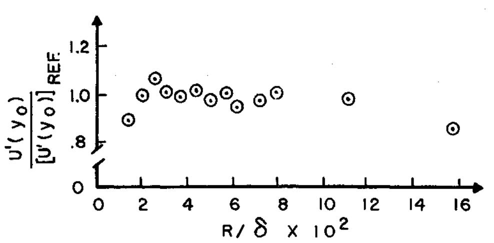

Fig. 3 Effect of R/δ on $u'(y_0)$

R, then the velocity gradient u' and the second derivative of the velocity u'' could be determined directly.

The relationship between the power P dissipated by the hot wire and the mean flow velocity V past the wire is the well-known[1] King's Law relationship $P = [A + B(V)^{1/2}]\Delta T$ where A and B are empirical constants and ΔT is the temperature difference between the sensor and its surroundings. Substituting Eq. (3) into King'slaw and expanding in a Binomial series

$$P(t) = [A + B(u^{1/2} + \{(R/2)u'/u^{1/2} + (R^3/16)(u'''/u^{1/2} + 3u'u''/u^{3/2})\} \sin \omega t + \{(R^2/8)(-u''/u^{1/2} - u'^2/u^{3/2} + \omega^2/u^{3/2})\} \cos 2\omega t + \{-(R^3/48)(u'''/u^{1/2} + 3u'u''/u^{3/2})\} \sin 3\omega t + \dots)]\Delta T \quad (7)$$

When the parameter R/δ is sufficiently small, the coefficient of $\sin \omega t$ is $BR\Delta T u'/2(u)^{1/2}$ and a direct determination of the velocity gradient can be made provided that R, B, ΔT and u are known.

From the preceding analysis, the parameters of interest in this study are R/δ and $\omega R/u(x_0, y_0)$. Both of these parameters should be kept as small as possible in order to provide a localized measurement with the minimum flow disturbance.

Experiment

An experiment was performed to investigate the response of an oscillating hot wire anemometer. A schematic of the flow system is given in Fig. 1. The shear layer formed in the wake of a two-dimensional wedge was investigated. The freestream Reynolds Number based on wedge thickness is 8000. The orientation of the mean flow velocity $u(x, y)$, the direction of oscillation R and the axis of the hot wire lie on the axes of the orthogonal coordinate system. The experiment is conducted by oscillating the probe and slowly moving the wedge such that the probe is exposed to the shear layer in the wake of the wedge.

A schematic of the electronics is given in Fig. 1. The anemometer has an output voltage signal which is proportional to the power P dissipated in the wire. At point 1 in Fig. 1 the anemometer output is expressed by Eq. (7). The Lock-In Amplifier, Princeton Applied Research Model 121, can be used to examine the coefficient of $\sin \omega t$ and $\cos 2\omega t$ in Eq. (7). Phase syncronization of the Lock-In Amplifier is provided by driving it at the frequency of forced oscillation from the shaker system. When the LIA is tuned to the frequency of forced oscillation ω, its output will be a d.c. voltage equal to the rms value of the coefficient of $\sin \omega t$ in equation 7, and when the amplifier is tuned to 2ω its output will be the rms amplitude on the coefficient of $\cos 2\omega t$ in Eq. (7). Because of system limitations, the experiment was operated at a frequency of 55 Hz with a range of amplitude 0.0008 in. $\leqq 2R \leqq 0.10$ in. A LIA time constant of 1 sec was found to be compatible with the speed at which the flow could be traversed.

Further details of the experiment, including the construction of special hot-wire probes and the feedback control of the shaker, are given in Ref. 2.

Results

Figure 2, trace A, is an x-y recorder plot of the shear layer velocity profile $(u)^{1/2}$ in the wake of the wedge as determined by the static operation of the hot-wire anemometer. The length scale, δ, of the gradient, was taken as the maximum slope thickness of the shear layer and was 3.16×10^{-2} in. Trace B is the output of the LIA when it is tuned to frequency ω; it represents $(R/2)B\Delta T\, u'(y)/(u)^{1/2}$ vs y. Trace C is the output of the LIA when tuned to frequency 2ω; it represents the coefficient of $\cos 2\omega t$ in Eq. (7).

To investigate the effect of R/δ on the determination of the gradient $u'(y)$, a series of tests was performed at different amplitudes of oscillation R. In each case the peak value of the slope $u'(y_0)$ was measured from trace B. The results are plotted on Fig. 3 as $u'/u'_{\text{ref.}}$ vs R/δ. The value of $u'/u'_{\text{ref.}}$ was arbitrarily set equal to 1 at $R/\delta = 3.8 \times 10^{-2}$. At the smallest value of R/δ on Fig. 3, the signal was being obscured by the noise level of the Hot Wire Anemometer system. As R/δ is increased, the data indicate a fairly constant value of measured velocity gradient up to $R/\delta \simeq 11.2 \times 10^{-2}$. For large values of R/δ, the measured gradient decreases because the amplitude of oscillation is so large that a localized measurement is no longer being made. The mean value of the measured gradient u' from Fig. 3 for $3 \times 10^{-2} \leq R/\delta \leq 11.2 \times 10^{-2}$ was 2.50×10^4 sec^{-1} while the graphically determined gradient was 2.18×10^4 sec^{-1}.

Since R/δ is on the order of 5×10^{-2} in this experiment, it is a valid approximation to neglect terms of order $(R/\delta)^3$ compared to terms of order (R/δ) in Eq. (7).

The qualitative appearance of the curve of trace C in Fig. 2 indicates that it is of the correct shape to be proportional to the second derivative of the velocity profile. However, investigation showed that the data taken with the LIA operating at frequency 2ω were deteriorated due to the relatively low level of the signal compared to the noise level of the anemometer electrics. Thus, no quantitative investigation of the data for the coefficient of $\cos 2\omega t$ was made.

Conclusion

It has been shown that it is possible to directly measure the velocity gradient in a fluid flow. The experiment has been performed by detecting the harmonic response of an oscillating hot-wire anemometer with a Lock-In Amplifier. Further work on improving the signal–to–noise ratio is necessary before the second derivative can be quantitatively determined.

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